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A NASTRAN Primer for the Analysis of Rotating Flexible Blades

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SUMMARY

This primer provides documentation for using MSC NASTRAN in analyzing rotating flexible blades. The analysis of these blades includes geometrically nonlinear (large displacement) analysis under centrifugal loading, and frequency and mode shape (normal modes) determination. The geometrically nonlinear analysis using NASTRAN Solution sequence 64 is discussed along with the determination of frequencies and mode shapes using Solution Sequence 63. A sample problem with the complete NASTRAN input data is included. Items unique to rotating blade analyses, such as setting angle and centrifugal softening effects are emphasized.

1.0 INTRODUCTION

The purpose of this primer is to document the use of MSC NASTRAN in analyzing rotating flexible blades. The analysis of rotating flexible blades such as compressor and turboprop blades, often requires complex procedures including geometrically nonlinear (large displacement) analysis and frequency and mode shape determination. The objective in performing such analyses includes the prediction of steady state deflections and stresses under centrifugal forces, the generation of data for constructing Campbell diagrams (plots of frequency versus rotational speed), and the provision of modal data for use in flutter calculations. In performing these analyses, and in modeling the complex geometries and material properties of the blades (fig. 1), finite element (F.E.) computer programs typically are used. NASTRAN is particularly well-suited because of its ability to compute steady-state displacements from its geometrically nonlinear analysis capabilities, and then, to use those results for a subsequent normal modes analyses.

This primer also can be used to bridge the gap between the theoretical modeling tactics presented in (ref. 1) for creating the model, and the practical application of NASTRAN for large displacement or normal modes analysis. Reference 1 provides a complete discussion of blade modeling strategies and documents the capabilities and limitations of various NASTRAN elements. The performance of a broad range of mesh configurations and element types also are evaluated for different blade related parameters such as camber, twist, sweep, and rotational speed.

The computation of steady-state displacements, frequencies, and modes shapes of flexible rotating blades requires that two NASTRAN Solution sequences

be run. First, a large displacement analysis is run using NASTRAN Solution Sequence 64. This solution sequence performs large displacement analysis on the rotating blade, computes steady-state displacements and stresses (due to the rotational effects), and then stores the blade's final stiffness and mass matrix in a database. Experience has shown (ref. 2) that a large displacement analysis is required because the blades are relatively flexible and normally deflect considerably under the centrifugal forces (fig. 2).

Following the large displacement analysis, the frequencies and mode shapes are computed using Solution Sequence 63. (A typical plot demonstrating the variation in natural frequencies with rotational speed is shown in fig. 3.) This solution sequence computes the modal parameters from the final mass and stiffness matrices which were computed during the Solution 64 run. The structural property matrices that correspond to the blade in its deformed position must be used so that the effects of centrifugal stiffening, and other elements which will be discussed, are included in the normal modes analysis.

2.0 DESCRIPTION OF SAMPLE PROBLEM

The sample problem in figure 4 is provided in order to demonstrate the procedures required for large displacement and normal modes analyses. All of the requirements for these analyses are exhibited in this sample problem even though this problem does not have the mesh complexity required of typical flexible blades. The mesh used in this sample problem is for demonstration purposes and is over simplified compared to typical analyses. Although there are no aerodynamic loads included in this problem, they can be included, and would be combined with the centrifugal loads for computing the steady-state displacements.

The sample problem consists of a rotating, swept back, flat plate. The plate is modeled with three plate elements (CQUAD4) connected by 8 grid points. The grid points are defined in a local rectangular coordinate system, which is rotated 30° from the axis of rotation in the X/Y plane (see section 6.0 for a discussion on the CORD2R card). It should be noted that the selection of the orientation of axes needs to be consistent with the directions that are specified for the centrifugal softening terms and the rotational speed (section 3.0).

3.0 GEOMETRICALLY NONLINEAR ANALYSIS/MSC NASTRAN SOLUTION 64

This section provides a discussion regarding the construction of the Solution 64 input data deck. The sample data deck is comprised of four components; the CRAY Job Control Language, the NASTRAN Executive, the Case Control, and Bulk Data, and is included in appendix A. The following sections discuss step by step the data cards included in the appendix.

3.1 CRAY Job Control Language

The CRAY Job Control Language for submitting jobs to the CRAY (at NASA Lewis Research Center) and running NASTRAN is given at the beginning of the data deck. The amount of time and memory indicated on the "JOB" card is based

on the number of degrees of freedom used for the model and the number of iterations (section 3.3) specified for the nonlinear analysis. Faster turnaround time on the computer is accomplished when these allocations are minimized.

On the "NASTRAN" card (see ref. 3 for a complete description), a temporary database named "BLADE" is specified. This database is used for storing the mass and stiffness matrices for subsequent use in the Solution 63 normal modes analysis.

3.2 Solution 64 Executive

Solution 64 is well suited to the large steady-state displacement analysis of rotating structures except for the fact that coriolis forces and centrifugal softening terms (see appendix C for a derivation of centrifugal softening) are not automatically included. Since the coriolis forces are velocity related, they do not have any influence on the large displacement analysis and do not need to be accounted for. Furthermore, from previous studies it has been determined that coriolis forces have negligible effects on thin, flexible blade frequencies (ref. 4), so they normally do not need to be included in the Solution 63 analysis. However, the centrifugal softening terms do need to be included in the analysis and must be added via DMAP (Direct Matrix) programming.

The centrifugal softening terms ($-w^2M$) are added to the global stiffness matrix for all grid points for the translational degrees of freedom in the direction of the two axes perpendicular to the axis of rotation. The softening terms are input into the Solution 64 analysis using the NASTRAN DMAP included with the sample problem given in appendix A. In DMAP Alter 77, the $-w^2$ coefficient first is computed from the rotational speed specified on the RPM parameter card which is provided in the bulk data deck (see section 3.4). This coefficient then is added into the matrix "K1OM". All terms in this matrix are zero except for the diagonal elements corresponding to the two translational degrees of freedom perpendicular to the axis of rotation. These elements are set equal to $-w^2$. The matrix "K1" which specifies the two perpendicular axes is defined using DMI cards in the bulk data deck (section 3.4). The K1OM matrix is multiplied by the global mass matrix to form the $-w^2M$ softening matrix and then the resulting matrix, KSOFT, is stored in a database. In alter 188 KSOFT is retrieved from the database ("DBFETCH..."), and is added to the global stiffness matrix (KJJ) in every iteration that takes place in the large displacement analysis.

3.3 Solution 64 Case Control

The cards in the CASE CONTROL Deck are used to specify the problem titles, the type of printed output, and the number or iterations that are to be carried out in the large displacement analysis.

For the sample problem the applied forces at all of the grid points (OLOAD=ALL), and the resulting displacements (DISP=ALL) and reactions (SPCFO=ALL), are printed in the last three iterations. It is recommended that displacements and reactions be printed in at least the last few of iterations so that convergence can be monitored. It may be desirable to print displacements in the first iteration so that the nonlinearity of the blade's response

can be assessed. Acceptable convergence is achieved when both the displacement changes between iterations and the force unbalance between the applied centrifugal forces and the internal element forces are small. The force unbalance at each of the unconstrained grid points is printed along with the reactions at the constrained points by using the command "SPCFO=ALL".

There are eight iterations, or "subcases," specified for this sample problem. In the first subcase a linear analysis is performed. In the second subcase, the displacements from the first subcase are used to form a differential stiffness matrix which is then used to compute a new set of displacements. In subsequent subcases the differential stiffness matrix is updated using the resultant displacements and a new set of displacements is computed. (Details of the theory underlying the iterative procedure and results for a flexible turboprop blade are provided in (refs. 2 and 5).)

When actual blades having large numbers of degrees of freedom are analyzed the cost of running a large number of iterations can be significant. To minimize the CPU time and cost, the number of iterations should be kept to a minimum. The best way to optimize the number of iterations is to specify a minimum number of iterations, and then if the solution has not converged, use NASTRAN "restart" (ref. 6) to resume the analysis.

3.4 Solution 64 Bulk Data Deck

The primary function of the bulk data deck is to describe the blade geometry, boundary conditions, material properties, and loads. Details of blade modeling techniques and the bulk data cards required for the model description are presented in (ref. 1), and in the NASTRAN User's Manual (ref. 6). In addition to describing the model, the bulk data deck is used to specify the rotational speed (RFORCE and PARAM RPM), and the matrix "K1" discussed in section 3.2.

NASTRAN automatically computes a centrifugal force field whenever an "RFORCE" card is used to specify a rotational speed. The centrifugal force field is computed by using the blade's geometrical and mass properties defined in the bulk data deck, and the rotational speed specified on the "RFORCE" card. The rotational speed is also included on a parameter card, "PARAM RPM." This card is used by the DMAP Alters for computing the value $-w^2$ for the centrifugal softening terms.

The matrix K1 is set up using DMI Bulk Data cards. In the first DMI card, the matrix is defined as being symmetric and of size 6 by 6. The size of this matrix is always 6 by 6 corresponding to the number of degrees of freedom at a grid point. In the next two DMI cards the elements of the matrix are defined as zero except for the diagonal terms corresponding to the two translational directions perpendicular to the axis of rotation which are set equal to 1.0. For this problem the Y and Z axis are the axes that are perpendicular to the axis of rotation. In alter 77 (section 3.2) the K1 matrix will be expanded so that the softening terms are applied at every grid point in the model.

Several issues which concern the blade model and are addressed in (refs. 1 and 7), are discussed below as they are relevant to the large displacement analysis of flexible blades. The first issue concerns the method of formulation

for the element mass matrices. Most of the elements available in NASTRAN permit the user to utilize a lumped or consistent mass matrix, but since the formulation for the centrifugal softening terms, and the centrifugal force field, is based on a lumped mass representation, a lumped mass matrix should also be used for the elements. Furthermore, no clear advantages have been found for utilizing a consistent mass formulation. A lumped mass matrix is computed by default in NASTRAN.

The next issue regards the lack of stiffness in the in-plane, normal rotation for plate and shell elements. This condition can present problems when adjacent elements lie in the same plane (coplanar). Since the elements have zero computed stiffness in the normal rotation, and the elements are coplanar, the accumulated rotational stiffness may end up being very close to zero. When this occurs the stiffness matrix becomes singular and the analysis fails. To circumvent this problem, these "small" rotational stiffnesses can be constrained with SPC's. This solution is sensible since the normal rotational stiffness usually is relatively stiff, thus it is reasonable to fully constrain the rotation. For nonlinear problems this approach is not feasible because the large displacements may deflect the elements such that elements that start out noncoplanar become coplanar. When this happens, singularities that did not exist in earlier iterations arise, and the solution fails in the resultant iteration. A feature in NASTRAN for overcoming this problem is the K6ROT parameter. This parameter adds artificial rotational stiffness at the element level so that even if elements are coplanar, the global stiffness matrix will not have any singularities. The effect of adding this rotational stiffness produces results similar to when the rotation is fully constrained with SPC's (see ref. 1 for details of comparison). The advantage of using the K6ROT parameter over using SPC's is that it is difficult to determine beforehand where SPC constraints are required.

The final issue concerns the constraint at the base of the blade model. For most blade analyses performed thus far, the base of the blade has been fully constrained. In reference 7, it was shown that base flexibility can have a significant effect on steady-state displacements, frequencies, and mode shapes. Therefore, whenever there is information on the blade's base support flexibility, it should be incorporated into the blade model.

4.0 NORMAL MODES ANALYSIS/MSC NASTRAN SOLUTION 63

The NASTRAN Solution 63 data deck for the normal modes analysis is given in appendix B. This data deck uses the identical model description that was used in the Solution 64 deck. The Solution 63 deck was created by duplicating all of data cards used in the Solution 64 deck for defining the blade model, removing the cards associated with the large displacement analysis, and then adding the necessary cards for the Solution 63 normal modes analysis.

4.1 CRAY Job Control Language

The amount of time and memory specified for the Solution 63 run is based on the number of degrees of freedom used in the analysis and the number of modes requested. As with the Solution 64 analysis, faster turnaround time on

the computer will be accomplished when the time and memory allocations are minimized. It also should be noted that the "NASTRAN" command accesses the same database ("BLADE") in the Solution 63 run as in the Solution 64 run.

4.2 Solution 63 Executive

The primary function of the Executive Control Deck is to request that Solution Sequence 63 be run (SOL 63).

4.3 Solution 63 Case Control

The Case Control deck specifies that the mass and stiffness matrices are to be retrieved off of the database (SEKR and SEMR), and that the method to be used for the eigensolution is on "EIGR" card 10 in the bulk data deck (METHOD=10). The "DISP=ALL" card specifies that the values of the computed mode shapes at all of the grid points are to be printed out.

4.4 Solution 63 Bulk Data Deck

The model description provided in the Solution 63 bulk data deck is not used for creating a new mass and stiffness matrix since these matrices are obtained from the database. These cards are required in the Data Deck for bookkeeping purposes.

The "EIGR" Card is used to specify the method for extracting the blade's frequencies and mode shapes. For this sample problem the inverse power method is chosen. This method of solution is recommended for flexible blades with a large number of degrees of freedom, and when there is interest only in the first few modes. For this blade, the frequency range of interest is specified to be between 50 and 3000 Hz with an estimate that there will be 7 modes in this range. The analyst must use care with these specifications because, if the range is made too large, there may be more modes in the range than was estimated and some of the modes may be missed, or the modes that are computed may be computed inaccurately. Conversely, if the range is made too small some of the modes may be outside of the range and not computed at all. In section 3.1.2 of the NASTRAN USER'S MANUAL (ref. 6) a description is given of the eigensolution summary which is printed along with the NASTRAN output when the inverse power method is used. This summary provides the analyst with information concerning the adequacy of the eigensolution and the reason for solution termination. The third reason for termination listed in section 3.1.2 (indicated by "All eigenvalues found in the frequency range specified") is most desired as it indicates that the solution was completed correctly. If this is not the reason for termination, the frequency range or the estimated number of modes in the range specified on the EIGR card likely will need to be adjusted.

5.0 NONROTATING BLADE NORMAL MODES ANALYSIS

The data deck for the Solution 63 analysis can be used when it is necessary to obtain the nonrotating blade frequencies and mode shapes (NASTRAN Solution Sequence 3 can also be used). When utilizing nonrotating blades in

the normal modes analysis SELA = 1 and the SEMA = 1 in the Solution 63 case control deck need to be changed to SELA = all and SEMA = all. When these changes are made, NASTRAN computes a new mass and stiffness matrix instead of pulling the mass and stiffness matrices off of the database which is done when SELA and SEMA are set to 1. Thus, the rotational effects computed in the large displacement analysis are not included in the property matrices since they are computed from data supplied for the nonrotating blade.

6.0 CHANGES IN ROTATIONAL SPEED

To obtain the steady state displacements, frequencies, and modes shapes when the blade is spinning at a new speed, both the large displacement and normal modes analyses must be rerun. The large displacement analysis (Solution 64) needs to be rerun because the steady-state position of the blade changes with rotational speed. Furthermore, updated mass and stiffness matrices, which reflect the effects of the current rotational speed, need to be transferred to the normal modes analysis performed in Solution 63. The effect of rotational speed on blade frequencies for a typical blade was shown in figure 3.

In the Solution 64 run the rotational speed is altered by changing the speed on the RFORCE and PARAM RPM cards. In addition to changing the blade's speed, the blade's angle of attack normally has to be adjusted when the blade is operating at a new rotational speed. Once the correct angle of attack is determined, either the axis that the blade is rotating about must be changed or the blade itself must be rotated. Due to the way the centrifugal softening terms are applied it is simpler to rotate the blade than to change the axis of rotation. To implement a change in the angle of attack, the entire blade can be rotated by defining the blade's geometry in a new coordinate system. This method of rotating the blade is convenient because the coordinates of the blade on the "GRID" cards do not have to be changed. Instead, the coordinate system in which the blade is described is rotated. Referring to figure 5, the blade cross section shown is rotated an angle α by prescribing that the blades coordinates are in reference to the X', Y', Z' system instead of from the original X, Y, Z system. The X', Y', Z' system is defined by using the "CORD2R" card which is described in section 7.3.2 of the NASTRAN Primer (ref. 8). The blade's grid points are designated to be in the primed coordinate system by changing the identification number of the coordinate system using a GRDSET bulk data card. It should be noted that all other items, such as the blade's rotational speed, continue to be in reference to the original, basic coordinate system.

7.0 COMBINED SOLUTION 64/SOLUTION 63 ANALYSIS

A combined large displacement, frequency, and mode shape analysis can be performed in the Solution 64 Sequence. This analysis is completed by adding NASTRAN DMAP Alters to the Solution 64 data deck (see appendix D). The function of these DMAP alters is to access the eigenvalue extraction routines that solve for the frequencies and mode shapes. The alters are setup so that the final mass and stiffness matrices generated in the large displacement analysis are used. The advantages of utilizing the combined analysis capability are reduced CPU, faster turnaround time, and reduced quantities of output listings. In general, the CPU time can be reduced by one half.

APPENDIX A

MSC/NASTRAN SOLUTION 64 INPUT DATA

```
$ CRAY JOB CONTROL LANGUAGE (JCL)


  JOB,JN=BLADE64,T=100,MFL=450000.
  ACCOUNT,AC=XXXXX, APW= XXXXX.
  NASTRAN,ID=XXXXX,DBASE=BLADE,FL=400000,STORE=TPOOL.
  /EOF

$ NASTRAN EXECUTIVE CONTROL DECK

  ID BLADE,BLADE
  APP DISP
  SOL 64
  TIME 100
  $
  $ DMAP ALTER FOR COMPUTING CENTRIFUGAL SOFTENING MATRIX
  $
  ALTER 77
  PARAMR   //MPY/V,N,OMEGA/V,Y,RPM/.1047198 $
  PARAMR   //MPY/V,N,OMEGASQ/OMEGA/OMEGA $
  PARAMR   //COMPLEX//OMEGASQ/0.0/V,N,ALPHAC $
  MATMOD   K1,,,,,/K1BOB,/5/LUSET $
  ADD      K1BOB,/K1OM/ALPHAC $
  MPYAD    K1OM,MJJ,/KSOFT/0/-1///1/6 $
  DBSTORE  KSOFT,//MODEL/0/ $
  $
  $  DMAP  ALTER FOR ADDING SOFTENING MATRIX TO STIFFNESS MATRIX
  $
  ALTER 188     $
  DBFETCH  /HKSOFT,,,,/MODEL/0/1 $
  ADD      KJJ,HKSOFT/KTOT// $
  EQUIV    KTOT,KJJ/ALWAYS $
  ENDALTER
  CEND
```

```
$ NASTRAN CASE CONTROL DECK


  TITLE = SWEPT, CANTILEVER BLADE
  SUBTITLE = LARGE DISPLACEMENT ANALYSIS AT 8350 RPM
     LOAD = 50
  SUBCASE 1
     LABEL = LINEAR ANALYSIS
  SUBCASE 2
     LABEL = DIFFERENTIAL STIFFNESS
  SUBCASE 3
     LABEL = SUBCASE 3
  SUBCASE 4
     LABEL = SUBCASE 4
  SUBCASE 5
     LABEL = SUBCASE 5
  SUBCASE 6
     LABEL = SUBCASE 6
     DISP = ALL
     OLOAD = ALL
     SPCFO = ALL
  SUBCASE 7
     LABEL = SUBCASE 8
     DISP = ALL
     OLOAD = ALL
     SPCFO = ALL
  SUBCASE 8
     LABEL = SUBCASE 10
     DISP = ALL
     OLOAD = ALL
     SPCFO = ALL
  BEGIN BULK


$ NASTRAN BULK DATA DECK


CORD2R  33      0       0.0     0.0     0.0     0.0     0.0
1.0     +2
+2      -.966   0.50    0.0

CQUAD4  1       10      1       2       4       3
CQUAD4  2       10      3       4       6       5
CQUAD4  3       10      5       6       8       7

DMI     K1      0       6       1       1               6       6
DMI     K1      2       2       1.0
DMI     K1      3       3       1.0
```

```
GRID     1        33      -1.6      0.0      5.9
GRID     2        33       0.4      0.0      6.1
GRID     3        33      -1.4      0.0      3.9
GRID     4        33       0.6      0.0      4.1
GRID     5        33      -1.2      0.0      1.9
GRID     6        33       0.8      0.0      2.1
GRID     7        33      -1.0      0.0     -.1                 123456
GRID     8        33       1.0      0.0      .1                 123456

PSHELL   10       10       0.10     10
MAT1     10       1.0+7             0.30     2.6-4

PARAM    MAXRATIO2.0+15
PARAM    RPM      8350.
PARAM    K6ROT    1.0
RFORCE   50       0                 139.2    1.0      0.0      0.0

ENDDATA
/EOF
```

APPENDIX B

MSC/NASTRAN SOLUTION 63 INPUT DATA

```
$ CRAY JOB CONTROL LANGUAGE (JCL)


  JOB,JN=BLADE64,T=100,MFL=450000.
  ACCOUNT,AC=XXXXX, APW= XXXXX.
  NASTRAN,ID=XXXXX,DBASE=BLADE,ACCESS=TPOOL.
  /EOF

$ NASTRAN EXECUTIVE CONTROL DECK

  ID BLADE,BLADE
  APP DISP
  SOL 63
  TIME 100
  CEND


$ NASTRAN CASE CONTROL DECK


  TITLE = SWEPT, CANTILEVER BLADE
  SUBTITLE = FREQUENCY ANALYSIS AT 8350 RPM
     SET 1 = 0
     SEKR = 1
     SEMR = 1
     METHOD = 10
     DISP = ALL
  BEGIN BULK


$ NASTRAN BULK DATA DECK


CORD2R  33      0       0.0     0.0     0.0     0.0     0.0
1.0     +2
+2      -.866   0.50    0.0

CQUAD4  1       10      1       2       4       3
CQUAD4  2       10      3       4       6       5
CQUAD4  3       10      5       6       8       7

DMI     K1      0       6       1       1               6       6
DMI     K1      2       2       1.0
DMI     K1      3       3       1.0
```

```
GRID     1        33      -1.6      0.0     5.9
GRID     2        33       0.4      0.0     6.1
GRID     3        33      -1.4      0.0     3.9
GRID     4        33       0.6      0.0     4.1
GRID     5        33      -1.2      0.0     1.9
GRID     6        33       0.8      0.0     2.1
GRID     7        33      -1.0      0.0     -.1              123456
GRID     8        33       1.0      0.0      .1              123456

PSHELL   10       10       0.10     10
MAT1     10       1.0+7             0.30    2.6-4

EIGR     10       INV      50.      3000.   7        7                      +E1
+E       MASS
ENDDATA
/EOF
```

APPENDIX C

CENTRIFUGAL SOFTENING

The equation of motion for a lumped mass, in a rotating reference frame can be written using (ref 9):

$$M\{\ddot{u}\} + 2\{\Omega\} \times \{\dot{u}\} + \{\Omega\} \times \{\Omega\} \times [\{R\} + \{u\}] = \{F\} \qquad (1)$$

In this equation, M is the mass, $\{\ddot{u}\}$, $\{\dot{u}\}$, and $\{u\}$ are the acceleration, velocity, and displacement vectors of the mass, $\{\Omega\}$ is the rotation vector, $\{R\} + \{u\}$ is the position vector from the mass to the rotational axis, and $\{F\}$ is the vector of applied forces.

If the rotation is about the Z axis then $\{\Omega\} = 0i + 0j + \Omega k$ and equation (1) can be written in matrix form as:

$$\begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix} \begin{Bmatrix} \ddot{u}_x \\ \ddot{u}_y \\ \ddot{u}_z \end{Bmatrix} + \begin{bmatrix} 0 & -2\Omega M & 0 \\ 2\Omega M & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{u}_x \\ \dot{u}_y \\ \dot{u}_z \end{Bmatrix} + \begin{bmatrix} k_x - \Omega^2 M & 0 & 0 \\ 0 & k_y - \Omega^2 M & 0 \\ 0 & 0 & k_z \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \\ u_z \end{Bmatrix} = \begin{Bmatrix} P_x + \Omega^2 M R_x \\ P_y + \Omega^2 M R_y \\ P_z \end{Bmatrix} \qquad (2)$$

The third term on the left hand side of this equation contains the centrifugal softening terms ($-\Omega^2 m$) in addition to the elastic stiffness terms. From equation (1) it can be seen that the softening terms arise from the displacement u of the mass about it's initial position. Since $\{u\}$ normally is much smaller than $\{R\}$, it is tempting to neglect the softening terms, however, Ω can be large, thus, for stability, large displacement, and normal modes analyses the softening terms are significant and cannot be neglected.

An alternate way of deriving the softening terms, is to consider the spring mass system subject to a centrifugal force $\Omega^2 \bar{R} M$ as shown in figure 6. The equilibrium equation for this system is written:

$$ku = \Omega^2 \bar{R} M \qquad (3)$$

where $\bar{R}$ is the distance from the mass to the stationary coordinate. Substituting $\bar{R} = R + u$ into the previous equation, the following is obtained:

$$ku = \Omega^2 (R + u) M$$

$$(k - \Omega^2 M) u = \Omega^2 R M \qquad (4)$$

This approach produces the same softening terms found previously in equation (2). From this formulation it is clear that the softening terms come from the fact that the centrifugal force is a function of both the initial position R and the displacement u. Furthermore, the part that is a function of u can be expressed as an increase in the centrifugal force or as a negative stiffness or softening term.

When performing the large displacement analysis, NASTRAN will automatically consider the increment in load by updating the centrifugal loads based

on the deformed position of the blade (PARAM,SKPLOAD). Although this approach will work for the large displacement analysis, the softening terms must still be inserted into the stiffness matrix when a subsequent normal modes analysis is to be performed. (In the sample problem both load updating and softening terms are used.) The softening terms must be included in the stiffness matrix that is transferred from the Solution 64 to the Solution 63 analysis, or the frequencies will be computed incorrectly.

To demonstrate the effect that the softening terms have on the natural frequencies, a steel plate (6" by 2" by 0.10") was analyzed (fig. 7). In this figure two sets of frequencies are plotted; one for the plate lying in the plane of rotation, and the other lying perpendicular to the plane of rotation. For the latter case the softening terms have a significant effect on the plate's first bending mode frequency. This is understandable since both the softening terms and the bending mode motion are in the plane of rotation. For actual blades, which have more complexity to their geometries than the steel plate, the centrifugal softening will have some influence on all of the modes and will therefore need to be included in all of the normal modes analyses.

APPENDIX D

COMBINED SOLUTION 64/63 DATA DECK

The following cards are added to the NASTRAN data deck for the combined analysis.

Executive Control Deck

```
ALTER 77    $
PARAM    //SUB/V,N,MSUBS/V,Y,NSUBS/1  $

ALTER 311   $
PARAM    //EQ/V,N,JEIG/MSUBS/NSKIP $
COND     LBL13,JEIG $

ALTER 315   $
DBFETCH  /DYNAMICS,MJJ,GM,,/MODEL/0/0 $
DBFETCH  /GKAA,,,,/SOLID/0/1 $
MATMOD   MJJ,,,,,/NVEC,/12/S,N,NULLS/2 $
EQUIV    MJJ,MNX/MPCF1 $
COND     LBLB3,MPCF1 $
MCE2     GUSET,GM,MJJ,,,/MNX,,, $
LABEL    LBLB3 $
COND     LBLB1,NULLS $
SCE1     GUSET,MNX,,,/MXBOB,,,,, $
JUMP     LBLB2 $
LABEL    LBLB1 $
EQUIV    MNX,MXBOB/ALWAYS $
LABEL    LBLB2 $
DPD      DYNAMICS,GPLS,SILS,USET,SLT,/GPLD,SILD,USETD,,,,,,,EED,
EQDYN/LUSET/V,N,LUSETD/V,N,NOTFL/V,N,NODLT/V,N,NOPSDL/
V,N,NOFRL/V,N,NONLFT/V,N,NOTRL/V,N,NOEED/C,N,0/V,N,NOUE $
READ     GKAA,MXBOB,,,EED,GUSET,CASECC/LAMA,VECTOR,
MI,OEIGS/MODES/S,N,NEIGS/NSUBS $
OFP      LAMA,OEIGS// $
COND     FIN,NEIGS $
SDR1     GUSET,,VECTOR,,,,GM,,,,/UGV,,QG/1/REIG $
SDR2     CASECC,FCSTMS,FMPT,FDIT,FEQEXINS,,FETT,,FBGPDT,LAMA,QG,
UGV,EST,XYCDB/OPG1,OQG1,OUGV1,OES1,OEF2,PUGV/REIGEN/-1 $
OFP      OUGV1,OPG1,OQG1,OEF2,OES1// $
LABEL    FIN $
PRTPARM  ///-1 $
ENDALTER
```

Case Control Deck

An additional subcase is added to the subcases required for the normal application of solution 64. This subcase contains the METHOD ID of the EIGR card in the BULK DATA deck.

```
SUBCASE XX  $ LAST SUBCASE
LABEL = EIGENVALUE EXTRACTION
METHOD = 22
DISP = ALL
```

Bulk Data Deck

In addition to the EIGR card a new parameter NSUBS must be defined as the total number of subcases in the CASE CONTROL deck.

```
PARAM,NSUBS,X   $$ TOTAL NUMBER OF SUBCASES
EIGR   22.................
```

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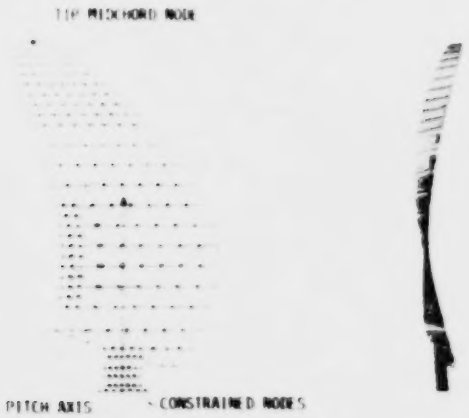

FIGURE 7 TYPICAL FLEXIBLE BLADE FINITE ELEMENT MODEL

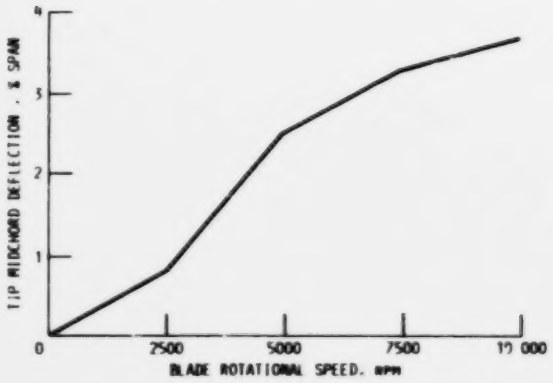

FIGURE 2. - TYPICAL NONLINEAR DEFLECTION CURVE FOR FLEXIBLE BLADE.

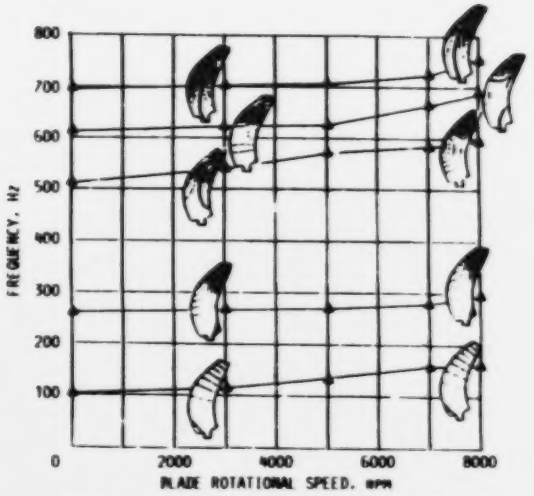

FIGURE 3. - TYPICAL FLEXIBLE BLADE CAMPBELL DIAGRAM.

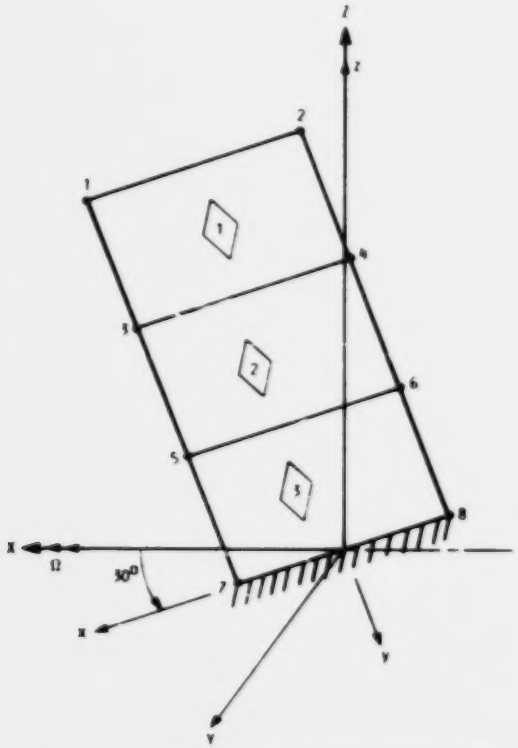

FIGURE 4. - DEMONSTRATION PROBLEM FINITE ELEMENT MODEL.

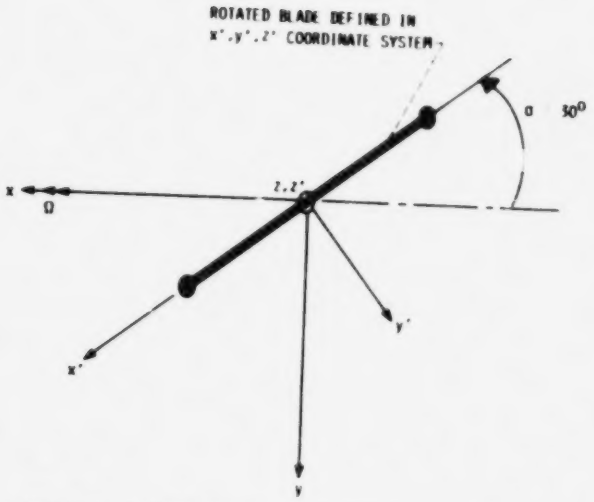

FIGURE 5. - ALTERING ANGLE OF ATTACK USING ROTATED COORDINATE SYSTEM.

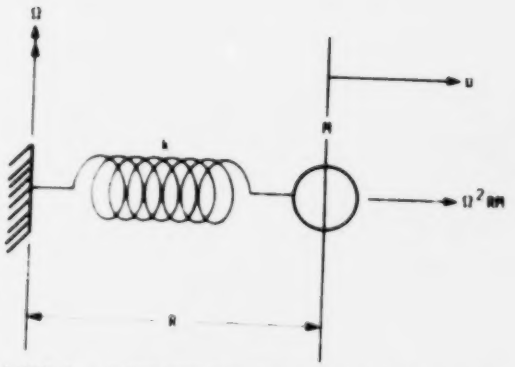

FIGURE 6. - SPRING MASS SYSTEM UNDER CENTRIFUGAL LOADING.

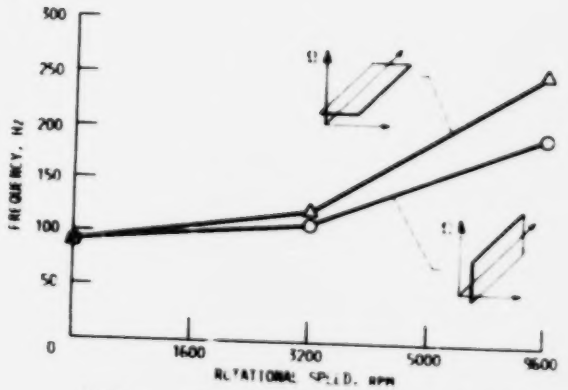

FIGURE 7. - EFFECT OF CENTRIFUGAL SOFTENING ON BLADE FREQUENCIES (FIRST BENDING FREQUENCIES).

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15. Supplementary Notes

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16. Abstract

This primer provides documentation for using MSC NASTRAN in analyzing rotating flexible blades. The analysis of these blades includes geometrically nonlinear (large displacement) analysis under centrifugal loading, and frequency and mode shape (normal modes) determination. The geometrically nonlinear analysis using NASTRAN Solution sequence 64 is discussed along with the determination of frequencies and mode shapes using Solution Sequence 63. A sample problem with the complete NASTRAN input data is included. Items unique to rotating blade analyses, such as setting angle and centrifugal softening effects are emphasized.

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